Question 1: The Upward-Sloping Aggregate Supply Curve

We consider a firm that produces output, Q_{it}, using a linear production technology in labor, which it hires at the wage rate W_t, of the form

$$Q_{it} = L_{it} \tag{1}$$

It sells its output for price P_{it} per unit of output. Its competitors charge P_t, which the firm can not affect. The total size of the market, also taken as given by the firm, is Y_t. Given the overall market size and price and the firm's price P_{it}, the demand for the firm's goods is

$$Q_{it} = (P_{it}/P_t)^{-\eta} Y_t \text{ , where } \eta > 1 \tag{2}$$

(a) If the firm increases its price by 1 percent, by how many percent does the demand for its good go down? What is the firm's own price elasticity of demand? (Hint: feel free to use a log-approximation for percentage changes)

Answer: If we take logarithms on both sides of the firm's demand equation we obtain that

$$\ln Q_{it} = -\eta \ln P_{it} + \eta \ln P_t + \ln Y_t.$$

Hence, we find that

$$\frac{\partial \ln Q_{it}}{\partial \ln P_{it}} = -\eta.$$

That is, when the firm increases its price by 1 percent, then its demand declines by η. η is the firm's own price elasticity of demand.

(b) What are the firm's real profits, i.e. its nominal profits divided by the price level P_t.

Answer: The firm's profits are given by the difference between its revenue and its production costs

$$P_{it}Q_{it} - W_t L_{it}.$$

Dividing this expression by the price level, we obtain that the firm's real profits equal

$$R_t = \frac{P_{it}}{P_t} Q_{it} - \frac{W_t}{P_t} L_{it}.$$

(c) What is the firm's level of marginal costs? Do they vary by its level of output?

Answer: No matter what its level of output, when the firm wants to produce one unit more of output it needs to higher one unit more of labor. The cost of this unit of labor is the wage rate W_t. Hence, the marginal cost of production for the firm are the wage W_t and do not depend on the level of output it produces.

(d) Solve for the firm's profit-maximizing price level. Show that its optimal price-setting decision boils down to charging a gross markup over marginal cost. Discuss how the markup depends on the firm's own price elasticity of demand.

Answer: By substituting the productiond and demand functions into the real profit function, we find that the firms chooses its price level, P_{it}, to maximize

$$R_t = \left(\frac{P_{it}}{P_t} - \frac{W_t}{P_t}\right)\left(\frac{P_{it}}{P_t}\right)^{-\eta} Y_t.$$

The first-order necessary condition associated with this profit maximization problem reads

$$0 = \frac{\partial R_t}{\partial P_{it}} = \left(\eta W_t - (\eta - 1) P_{it}\right)\left(\frac{P_{it}}{P_t}\right)^{-\eta} \frac{Y_t}{P_t}.$$

The solution to this condition yields that

$$P_{it} = \frac{\eta}{\eta - 1} W_t.$$

Hence, the firm will set its price equal to a (gross) markup, $\eta / (\eta - 1) > 1$, times its marginal cost of production. For the markup we find

$$\frac{\partial}{\partial \eta}\left(\frac{\eta}{\eta - 1}\right) = \frac{1}{\eta - 1}\left(1 - \frac{\eta}{\eta - 1}\right) < 0 \text{ and } \lim_{\eta \to \infty} \frac{\eta}{\eta - 1} = 1.$$

Thus the more elastic the demand for the good supplied by the firm the less market power it has and the lower the markup it charges over its marginal cost of production. In the limit, when the own price elasticity of demand for the firm goes to infinity, the firm will set its price equal to marginal cost, just like under perfect competition.

In the previous profit-maximization problem we have assumed that the firm can costlessly adjust its prices. Suppose the firm faces a quadratic adjustment cost to changing its price level and the cost parameter to this cost is $\xi > 0$. In that case, we derived in class that the firm's optimal pricing decision is

$$\pi_{it}\left(1 + \pi_{it}\right) = \left[(1 - \eta) P_{it} + \eta W_t\right] \frac{1}{\xi}\left(\frac{P_{it}}{P_t}\right)^{-\eta} \frac{Y_t}{P_t} + \frac{1}{1 + r_t} \pi_{it+1}\left(1 + \pi_{it+1}\right) \quad (3)$$

(e) Show that, if $\xi = 0$, this solution boils down to the one you derived in part (d).

Answer: This can be easiest seen by multiplying both sides of the above equation by the adjustment cost parameter ξ. This allows us to rewrite the optimal pricing decision as

$$\xi \pi_{it}\left(1 + \pi_{it}\right) = \left[\eta W_t - (\eta - 1) P_{it}\right]\left(\frac{P_{it}}{P_t}\right)^{-\eta} \frac{Y_t}{P_t} + \frac{\xi}{1 + r_t} \pi_{it+1}\left(1 + \pi_{it+1}\right).$$

If $\xi = 0$ then this reduces to

$$0 = \left[\eta W_t - (\eta - 1) P_{it}\right] \left(\frac{P_{it}}{P_t}\right)^{-\eta} \frac{Y_t}{P_t},$$

which is exactly de optimal pricing condition we derived in the case of flexible prices in part (d).

(f) In the symmetric equilibrium in which all individual firms behave identically, (3) reduces to (approximately).

$$\pi_t = \left[\frac{\eta}{(\eta - 1)} \frac{W_t}{P_t} - 1\right] \frac{\eta - 1}{\xi} Y_t + \frac{1}{1 + r_t} \pi_{t+1} \tag{4}$$

Derive this approximation.

Answer: In the symmetric equilibrium all firms set the same price, such that $P_{it} = P_t$ for all i and that thus $\pi_{it} = \pi_t$. This means that the optimal pricing condition simplifies to

$$\begin{aligned} \pi_t (1 + \pi_t) &= \left[\eta W_t - (\eta - 1) P_t\right] \frac{1}{\xi} \frac{Y_t}{P_t} + \frac{1}{1 + r_t} \pi_{t+1} (1 + \pi_{t+1}) \\ &= \left[\frac{\eta}{\eta - 1} \frac{W_t}{P_t} - 1\right] \frac{\eta - 1}{\xi} Y_t + \frac{1}{1 + r_t} \pi_{t+1} (1 + \pi_{t+1}). \end{aligned}$$

The final thing to realize is that for relatively low inflation rates $\pi_t (1 + \pi_t) \approx \pi_t$, such that we can approximate

$$\pi_t = \left[\frac{\eta}{\eta - 1} \frac{W_t}{P_t} - 1\right] \frac{\eta - 1}{\xi} Y_t + \frac{1}{1 + r_t} \pi_{t+1},$$

which is the expression we were asked to derive.

(g) Based on the analysis above, which price changes incorporate more of expectations about future inflation, those of tomatoes in the supermarket or those of magazine subscriptions. Explain.

Answer: According to (4), current inflation is determined by two factors. The first term on the right-hand side of the equation reflects current marginal costs. The second term reflects (expected) future inflation which the firm takes into account in its current price-setting decision in case adjusting prices is costly. As can be seen from (4), the more costly adjusting its prices are, i.e. the higher ξ, the lower the weight on marginal costs and the higher the relative weight on (expectations) of future inflation. Since adjusting the prices of tomatoes in the supermarket is much less costly than adjusting the prices of magazine subscriptions (many of which have been precommitted to) we would expect price changes of magazine subscriptions to be more influenced by expectations of future inflation than price increases of tomatoes.

Question 2 Debt-to-GDP ratio

In this problem, we use the government budget constraint to figure out what size of structural deficits might sustainable based on future economic growth. For this purpose we consider the government-debt accumulation equation

$$\underbrace{\overset{\bullet}{D}(t)}_{\text{change in government debt}} = \underbrace{\underbrace{[G(t) - T(t)]}_{\text{primary deficit}} + r(t)D(t)}_{\text{budget deficit}} \tag{5}$$

Here $D(t)$ is real government debt at time t, $G(t)$ real government purchases at time t, $T(t)$ is real net tax revenue at time t, and $r(t)$ real interest rate at time t.

The Government accumulates this debt subject to the No-Ponzi condition

$$\lim_{t\to\infty} e^{-\int_0^t r(s)ds} D(t) = 0 \tag{6}$$

(a) Suppose the government currently owes $D(0)$ and it is not expected to ever run a primary surplus, that is $G(t) > T(t)$ for all $t > 0$. Can this government ever satisfy the No-Ponzi condition? Why or why not?

Answer: No, a government with $D(0) > 0$ and $G(t) > T(t)$ for all $t > 0$ will violate the No-Ponzi condition. To see this, we can use the solution that

$$D(t) = \int_0^t e^{\int_s^t r(s)ds} (G(s) - T(s))\, ds + e^{\int_0^t r(s)ds} D(0),$$

such that

$$e^{-\int_0^t r(s)ds} D(t) = \int_0^t e^{-\int_0^s r(s)ds} (G(s) - T(s))\, ds + D(0).$$

Hence, if $G(t) - T(t) > 0$ for all $t > 0$ then it must be the case that

$$\lim_{t\to\infty} e^{-\int_0^t r(s)ds} D(t) > D(0) > 0,$$

which is a violation of the No-Ponzi condition. This is the case because the government never runs a primary surplus to pay off its interest and debt. In this case the government continuously finances it current debt by issuing new debt... A classic Ponzi scheme.

(a) We now consider the debt-to-GDP ratio $D(t)/Y(t)$. We denote the growth rates of GDP as $g(t) = \dot{Y}(t)/Y(t)$.

(b) Show that the debt to GDP ratio accumulates according to

$$\overset{\bullet}{\left(\frac{D(t)}{Y(t)}\right)} = \underbrace{\frac{G(t) - T(t) + r(t)D(t)}{Y(t)}}_{\text{Deficit as a fraction of GDP}} - g(t)\frac{D(t)}{Y(t)} \tag{7}$$

Answer: Applying the chain rule of derivatives yields that

$$\begin{aligned}
\overset{\bullet}{\left(\frac{D(t)}{Y(t)}\right)} &= \left(\frac{1}{Y(t)}\right)\overset{\bullet}{D}(t) - D(t)\frac{\overset{\bullet}{Y}(t)}{(Y(t))^2} \\
&= \left(\frac{1}{Y(t)}\right)\overset{\bullet}{D}(t) - \left(\frac{\overset{\bullet}{Y}(t)}{Y(t)}\right)\left(\frac{D(t)}{Y(t)}\right) \\
&= \left(\frac{1}{Y(t)}\right)\overset{\bullet}{D}(t) - g(t)\left(\frac{D(t)}{Y(t)}\right) \\
&= \frac{G(t) - T(t) + r(t)D(t)}{Y(t)} - g(t)\left(\frac{D(t)}{Y(t)}\right),
\end{aligned}$$

which is the equation.we are supposed to derive

(c) Suppose a country has a debt-to-GDP ratio of 160% and uncertainty about its ability to pay off its debt increases the (real) interest rate it pays on its debt by 1 percentage point (100 basis points). How much must $T(t)/Y(t)$ increase to raise the taxes needed to pay for the increased interest payments?

Answer: The increase in the debt payments would amount to $0.01 \times 160 = 1.6$ percentage points of GDP. Hence, to pay for the increase in the interest payments on its debt through tax revenue, the government would have to increase taxes by 1.6 percent of GDP.

For the rest of this question we consider a country that runs a structural deficit of 2 percent of GDP. That is

$$\frac{G(t) - T(t) + r(t)D(t)}{Y(t)} = 0.02 \text{ for all } t. \tag{8}$$

This is the maximum budget deficit allowed by the Stability and Growth Pact between members of the Eurozone. GDP in the country grows at a constant rate $g(t) = g$ for all t.

(d) What is the long-run steady-state debt-to-GDP ratio in that case? How does it depend on g?
(Hint: The steady-state debt-to-GDP ratio is that for which (7) is zero)

Answer: The steady state debt-to-GDP ratio satisfies

$$0 = \frac{G(t) - T(t) + r(t)D(t)}{Y(t)} - g(t)\left(\frac{\overline{D}}{Y}\right),$$

such that

$$\frac{\overline{D}}{Y} = \frac{1}{g}\left(\frac{G - T + rD}{Y}\right) = \frac{0.02}{g}$$

(e) At this steady state ratio and for a constant interest rate $r(t) = r$, what fraction of GDP does the country then need to pay in interest on its public debt?

Answer: The steady state interest burden as a fraction of GDP is

$$r\overline{\frac{D}{Y}} = 0.02\frac{r}{g}.$$

(f) How does this long-run interest burden depend on the growth rate g and interest rate r? What happens to this burden if a country with below average growth joins a monetary union whose equilibrium real interest rate is substantially higher than that in the country itself?

Answer: This burden is increasing in the interest rate because a higher interest rate leads to higher interest payment. It is decreasing in the growth rate of GDP. That is, if GDP grows relatively fast then previously accumulated debt relative to past GDP is smaller relative to current GDP. If a country with a relatively low growth rate g joins a monetary union that has a higher natural rate of interest, r, than the country itself, then the steady state interest burden as a fraction of GDP will increase because by joining the monetary union the country is now subjected to that union's r rather than its own.