Exam Advanced Machine Learning
7 January 2020, 18.30–21.15

This exam consists of 6 problems, each consisting of several questions. All answers should be motivated, including calculations, formulas used, etc. The use of a calculator is not allowed.

Question 1: Short questions
Please provide an argument for your answer on the following questions.

(a) Is the following statement true or false? A convolutional neural network (CNN) for image analysis can be trained for unsupervised learning tasks, whereas an ordinary neural network cannot.

(b) Is the following statement true or false? A perceptron model can achieve zero training error on *any* linearly separable dataset.

(c) How does the bias-variance decomposition of a ridge regression estimator compare with that of ordinary least squares regression?

(d) Is the following statement true or false? Logistic regression is equivalent to a neural network without hidden units and using the cross-entropy loss.

(e) What are some practical problems with the sigmoidal activation functions in neural networks?

Question 2: Neural networks

(a) Suppose that you build a neural network for a specific purpose. You train your network with cost function $J = \frac{1}{2}|\mathbf{y} - \mathbf{z}|^2$.

- When the input $\mathbf{x}$ is given to the network, first a weight matrix V mapping the input layer to the hidden layer is applied to yield $\mathbf{g} = V\mathbf{x}$.
- The vector of hidden unit values $\mathbf{g}$ are then activated using a ReLU activation function r, yielding $\mathbf{h} = r(\mathbf{g})$.
- Finally, the activated values are transformed to the output $\mathbf{z}$ by a weight matrix W given by $\mathbf{z} = W\mathbf{h}$.

Derive $\partial J/\partial W_{ij}$ and $\partial J/\partial V_{ij}$ for this network.

(b) Suppose that you have a 3-dimensional input $x = (x_1, x_2, x_3) = (2, 2, 1)$ fully connected to 1 neuron with activation function g_i. The forward propagation can be written as

$$a_i = g_i\left(w_1x_1 + w_2x_2 + w_3x_3 + b\right).$$

After training this network, the values of the weights and bias are $w = (w_1, w_2, w_3) = (0.5, -0.2, 0)$ and $b = 0.1$. You try 4 different activation functions $g_1, \ldots, g_4$, which respectively output the values $a_1 = 0.67$, $a_2 = 0.70$, $a_3 = 1.0$, and $a_4 = 0.70$. What is a valid guess for the activation functions $g_1, \ldots, g_4$? You can choose for each activation function from the set of sigmoid, tanh, linear, ReLU, leaky ReLU, and indicator functions.

(c) Explain what effect the following operations generally have on the bias and variance of your model.

(1) Regularizing the weights;

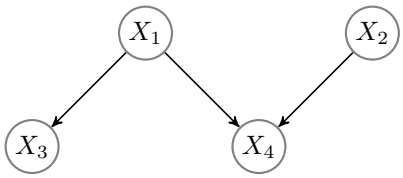
(2) Increasing the size of the layers (more hidden units per layer);

(3) Using dropout to train a deep neural network;

(4) Getting more training data (from the same distribution as before).

Question 3: Graphical models

The following figure shows a graphical model over four binary valued variables $X_1, \ldots, X_4$. We do not know the parameters of the probability distribution associated with the graph.



(a) Write the expression for the joint probability $\mathbb{P}(X_1, X_2, X_3, X_4)$ of the network in its *reduced* factored form.

(b) Would it typically help to know the value of X_3 so as to gain more information about X_2?

(c) Assume we already know the value of X_4. Woud it help in this case to know the value of X_3 to gain more information about X_2?

(d) List three different conditional independence statements between the four variables that can be inferred from the graph. You can include marginal independence by saying "given nothing".

Question 4: Hidden Markov Models (HMMs)

Consider a two-state hidden Markov model specified by (π, A, φ) that can output 4 possible values. Thus, the hidden states $z_i \in \{1, 2\}$, and the output values $x_i \in \{1, 2, 3, 4\}$. The further specification of the hidden Markov model is given as follows:

$$\pi = (0.5, 0.5), \qquad A = \begin{pmatrix} 0.99 & 0.01 \\ 0 & 1 \end{pmatrix}, \qquad \varphi = \begin{pmatrix} 0 & 0.199 & 0.8 & 0.001 \\ 0.1 & 0 & 0.7 & 0.2 \end{pmatrix}.$$

(a) Give an example of an output sequence of length 2 that cannot be generated by the hidden Markov model specified above.

(b) We generated a sequence of 2020^{2020} observations from the hidden Markov model, and found that the last observation in the sequence was 3. What is the most likely hidden state corresponding to that last observation?

(c) Consider an output sequence with a 3 followed by another 3. What is the most likely sequence of hidden states corresponding to these observations?

(d) Consider an output sequence with a 3 followed by another 3 followed by a 4. What are the *first two states* of the most likely sequence of hidden states corresponding to these observations?

Question 5: Reinforcement learning
We are using Q-learning to learn a policy in a system with two states s_1 and s_2. State s_1 has two actions (a and b), and state s_2 has only one action (c). Suppose that the discount factor is γ, and the learning rate is α. We initialize the Q-table with all zero values.

(a) On the first transition, you start in state s_1, apply action a, receive an immediate reward of 1, and then land in state s_2. What is the resulting Q-table? Give your answer as an algebraic expression that may include one or both of the symbols γ and α.

(b) On the second transition, you apply action c, receive an immediate reward of 0, and then land in state s_1. What is the resulting Q-table? Give your answer as an algebraic expression that may include one or both of the symbols γ and α.

(c) On the third transition, you apply action b, receive an immediate reward of 1, and then land in state s_2. What is the resulting Q-table? Give your answer as an algebraic expression that may include one or both of the symbols γ and α.

(d) What is the optimal policy that Q-learning has learned so far?

Question 6: Regularized linear regression
Recall that the objective function for the L_2 regularized linear regression is given by

$$J(\mathbf{w}) = ||X\mathbf{w} - \mathbf{y}||_2^2 + \lambda||\mathbf{w}||_2^2,$$

where X is the design matrix (the rows of X are the data points). During the lecture, we derived that the global minimizer of J is given by

$$\mathbf{w}^* = (X^T X + \lambda I)^{-1} X^T \mathbf{y}.$$

Now, let us consider running Newton's method to minimize J. Thus, let $\mathbf{w}_0$ be an arbitrary initial guess for Newton's method. One update step is then given by

$$\mathbf{w}_1 = \mathbf{w}_0 - [H(J(\mathbf{w}))]^{-1} \nabla_{\mathbf{w}} J(\mathbf{w}),$$

where H is the Hessian, and ∇ is the gradient.

(a) Show that $\mathbf{w}_1$, the value of the weights after one Newton step, is equal to $\mathbf{w}^*$.

partial grade	1	2	3	4	5	6
(a)	1	3	1	1	1	3
(b)	1	2	1	1	1	
(c)	1	2	1	1	1	
(d)	1		1	1	1	
(e)	1					

Final grade is: (sum of partial grades) / 9.0 + 1.0