Midterm Exam Advanced Logic 2023-2024

Thursday February 29, 2024, 18.45–21.00

If you use a bisimulation, you do not have to prove that it is a bisimulation. Motivate your answers.

Exercise 1. (2+2 points)
We work in a model $\mathcal{M} = ((W, R), V)$. A successor of a state x is a state $y \in W$ with Rxy, and a state x is blind if there is no $y \in W$ such that Rxy.

(a) Give a formula ϕ that is true in a state x if and only if x has at least one non-blind successor and at least one blind successor.

(b) Why is it not possible to give a formula that is true in a state x if and only if x has two different blind successors?

Exercise 2. (3+3 points)
Consider the formula $\phi = \Diamond p \to (\Box q \vee \Diamond(p \wedge \neg q))$.

(a) Prove or disprove universal validity of ϕ in words (no sequents, no tableaux).

(b) Investigate the validity of ϕ using sequents or tableaux. In case the formula is not valid, give (as a picture) a countermodel corresponding to your findings.

Exercise 3. (4 points)
A frame $\mathcal{F} = (W, R)$ is said to be reflexive if Rxx for all $x \in W$.
Prove that the formula $\psi = p \to \Diamond p$ characterizes reflexivity.

Exercise 4. (3 points)
Consider the frame $\mathcal{F}$ with set of states $W = \{0, 1\}$, and accessibility relation $R = \{(0,0), (0,1)\}$, and the valuation $V(p) = \{1\}$ on $\mathcal{F}$. Consider also the frame $\mathcal{F}'$ with set of states $W' = \{a, b, c, d\}$, and accessibility relation $R' = \{(a,b), (a,c), (c,c), (c,d)\}$, and the valuation $V'(p) = \{b\}$ on $\mathcal{F}'$.

Investigate whether the pointed models $\mathcal{F}, V, 0$ and $\mathcal{F}', V', a$ are bisimilar using the game approach.

The exam grade is $((\textit{number of points})/17) \times 9) + 1$