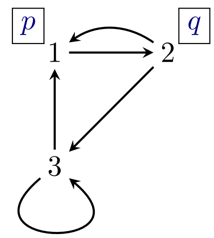
Exam Advanced Logic 2022-2023

Tuesday March 28, 2023, 08.30–10.45

8 exercises

If you use a bisimulation, you do not have to prove that it is a bisimulation.

Exercise 1. (*10 points*)
Consider the following model:



Give the complete game tree starting in state 3 with the formula $\Diamond\Diamond p \wedge \Diamond\Box q$, and indicate if possible a winning strategy for Verifier.

Exercise 2. (*10 points*)
Investigate using a tableau or sequents the universal validity of the formula

$$p \wedge \Diamond p \to \Diamond(p \wedge q) \vee \Box(\neg p \wedge \neg q)$$

In the case of non-validity, give your counterexample explicitly.
You do not have to give the preprocessing explicitly.

Exercise 3. (*10 points*)
Prove validity of $\Box p \to \Diamond\Box\Diamond p$ on the class of reflexive frames.
Recall: a frame (W, R) is reflexive if Rxx for all $x \in W$.

Exercise 4. (*10 points*)
A frame $\mathcal{F} = (W, R)$ is said to have property P if for all $x, y, z \in W$: if Rxy and Ryz then not Rxz.
Show that property P is not definable in basic modal logic.

Exercise 5. (*10 points*)
Consider the formulas $\phi = p \to \Box\Box p$ and $\psi = p \to \Box\Box\Box p$.
Give if possible a frame $\mathcal{F}$ is which ϕ is valid but ψ is not valid, and a frame $\mathcal{G}$ in which ψ is valid but ϕ is not valid. Prove only both invalidities.

Exercise 6. (*10 points*)
Show that $\Diamond\Box\Diamond p \to \Box\Diamond p$ is not derivable in system S4.

We recall the definitions of the Hilbert systems:

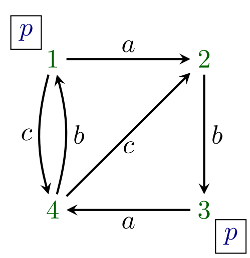
- system K is the most basic system with the tautologies of first-order propositional logic as axioms, the axiom for modal distribution ($\Box(p \to q) \to (\Box p \to \Box q)$), the rule for modus ponens, the rule for necessitation (if $\vdash \phi$ then $\vdash \Box\phi$), and the rule for substitution;
- system T is system K plus the truth axiom (or veridicality): $A1 : \Box p \to p$;
- system $S4$ is system T plus the axiom of positive introspection: $A2 : \Box p \to \Box\Box p$;
- system $S5$ is system $S4$ plus the axiom of negative introspection: $A3 : \neg\Box p \to \Box\neg\Box p$.

Exercise 7. (*10+10 points*)
Prove or disprove universal validity in PDL-models of the following:

(i) $[a^*]p \wedge [b^*]p \;\to\; [(a \cup b)^*]p$,

(ii) $[(a \cup b)c]p \;\to\; [ac \cup bc]p$.

Exercise 8. (*10 points*)
Consider the PDL-extension of the following $\{a, b, c\}$-frame.



First, compute the accessibility relation for $\alpha = \mathsf{while}\, p\, \mathsf{do}\, ab \cup c^*$. Give at least as intermediate steps the accessibility relation for $ab \cup c^*$ and for $(p?; ab \cup c^*)^*$.
Second, what are the states in which $p \to [\alpha]\neg p$ is true? (No proof needed.)

The exam grade is (the total number of points plus 10) divided by 10.
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