Midterm Exam Advanced Logic 2022-2023

Thursday March 2, 2023, 18.45–21.00

5 exercises

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*If you use a bisimulation, you do not have to prove that it is a bisimulation.*

**Exercise 1.** (2 points)
Give a formula $\phi$ that is true in a state $x$ if and only if $x$ does not have a blind successor.

We work in a model $\mathcal{M} = ((W, R), V)$. A successor of a state $a$ is a state $b \in W$ with $Rab$, and a state $a$ is blind if there is no $b \in W$ such that $Rab$.

**Exercise 2.** (4 points)
Prove or disprove in words (no sequents, no tableau) the universal validity of the following formula: $\Diamond(p \to q) \to (\Box p \to \Diamond q)$.

**Exercise 3.** (4 points)
Investigate using either a tableau or sequents the universal validity of the following formula: $p \wedge \Diamond p \ \to \ \Diamond(p \wedge \neg q) \ \vee \ \Box q$.

In the case of non-validity, give your counterexample explicitly.
You do not have to give the steps of the pre-processing.

**Exercise 4.** (5 points)
Consider the frame $\mathcal{F} = (W, R)$ with $W$ and $R$ given by

$$W = \{a, b, c, d\} \qquad R = \{(a, b), (b, c), (c, a), (d, a), (d, c)\}$$

and the model $\mathcal{M} = (\mathcal{F}, V)$ with valuation $V$ defined by $V(p) = \{a, c\}$.
Prove that $p \to \Box\Box\Box p$ is globally true in $\mathcal{M}$, but not valid in $\mathcal{F}$.

**Exercise 5.** (5 points)
A frame $(W, R)$ is said to be binary if it has the following property:
for every $x \in W$ there are $y, z \in W$ such that $Rxy$ and $Rxz$ and $y \neq z$.
So every state has at least two different successors.
Prove that 'being binary' is not definable by a formula in Basic Modal Logic.

*The exam grade is* $((\textit{number of points})/20) \times 9) + 1$