Exam Advanced Logic

VU University Amsterdam, 26 March 2013, 15:15–18:00

This exam consists of four questions. Use of textbook, definition sheets, etc. is not allowed. In total you can score 90 points as indicated per question. The final grade is the minimum of 10 and (points/10 + 1 + bonus).

---

1. (a) Define what it means that a modal formula is globally true in a model. *(3 pt)*

(b) Define what it means that a modal formula is valid in a frame. *(3 pt)*

Consider the frame $\mathfrak{F} = (W, R)$ with W and R given by

$$W = \{a, b, c, d\} \qquad R = \{(a, b), (b, c), (c, a), (d, a), (d, c)\}$$

and the model $\mathfrak{M} = (\mathfrak{F}, V)$ with valuation V defined by $V(p) = \{a, c\}$.

(c) Give a graphical representation of $\mathfrak{M}$. *(2 pt)*

(d) Prove that $p \to \Box\Box\Box p$ is globally true in $\mathfrak{M}$, but not valid in $\mathfrak{F}$. *(4+4 pt)*

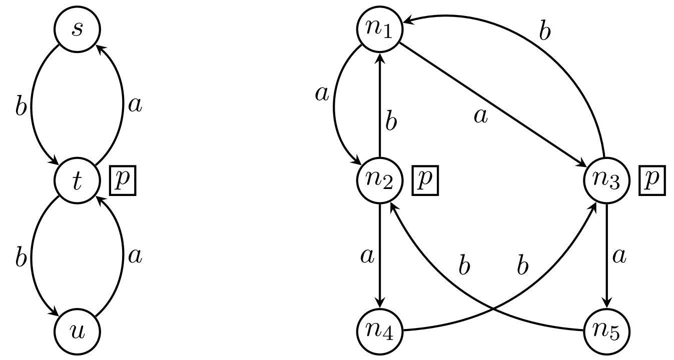
(e) Prove that for any formula φ, the formula $\Box\varphi \leftrightarrow \Box\Box\Box\Box\varphi$ is valid in $\mathfrak{F}$. *(8 pt)*

2. (a) Let I be an arbitrary index set, and let $i, j \in I$. Prove that the formula $p \to [i]\langle j\rangle p$ characterizes the class of I-frames $\mathfrak{F} = (W, \{R_k \mid k \in I\})$ that satisfy the property $R_i \subseteq R_j^{-1}$. *(10 pt)*

(b) Use the result of the previous question to show that the formula $\langle i\rangle[j]p \to p$ also characterizes the frame property $R_i \subseteq R_j^{-1}$. *(7 pt)*

(c) Are the formulas $p \to [i]\langle j\rangle p$ and $\langle i\rangle[j]p \to p$ equivalent? Prove your answer. *(8 pt)*

3. Consider the $\{a, b\}$-models $\mathcal{M}$ and $\mathcal{N}$ defined by:



$\mathcal{M}$ $\qquad\qquad$ $\mathcal{N}$

(a) Define model $\mathcal{M}$ by means of set notation. *(2 pt)*

(b) Is there a modal formula that distinguishes state n_3 in model $\mathcal{N}$ from state t in model $\mathcal{M}$? Prove your answer. *(10 pt)*

(c) Let $\widehat{\mathcal{N}}$ be the PDL-extension of model $\mathcal{N}$. Compute the transition relation $\widehat{R}_\pi$ corresponding to the PDL-program $\pi = \textsf{if } p \textsf{ then } ba \textsf{ else } ab$ *(8 pt)*

(d) Determine whether the PDL-formula $[b]\bot \to ([\pi]p \to \bot)$ globally holds in $\widehat{\mathcal{N}}$. Prove your answer. *(6 pt)*

4. System T is the extension of the minimal modal logic K with the axiom of veridicality (if something is known, it is true). System $S4$ extends T with the axiom of positive introspection; $S5$ extends $S4$ with the axiom of negative introspection.

Prove or disprove the following epistemic claims (you may use completeness theorems):

(a) $\vdash_T p \to \neg K \neg p$ *(5 pt)*

(b) $\vdash_{S4} q \vee K \neg K q$ *(5 pt)*

(c) $\vdash_{S5} \neg K K p \to K \neg K p$ *(5 pt)*