Advanced Econometrics Practice Exam

Master Econometrics and Operations Research
School of Business and Economics

Exam:	Advanced Econometrics (4.1)
Code:	E_EORM_AECTR
Examinator:	-
Co-reader:	-
Date:	-
Time:	-
Duration:	2 hours and 45 minutes
Calculator:	Not Allowed
Graphical calculator:	Not Allowed
Scrap paper:	Allowed
Number of questions:	3
Type of questions:	Multiple Choice and Open
Answer in:	English
Credit score:	100 credits counts for a 10
Grades:	-
Inspection:	-
Number of pages:	7, including front page

- Give justifications for your answers unless stated otherwise.
- Be complete and explicit, but also clear and concise in your statements.
- If you think that further information is needed to answer a question, or that the question is ill-posed, then explain your reasoning
- The questions should be handed back at the end of the exam. Do not take it home.

Good luck!

Question 1 [40 points] - Multiple Choice

For each of the following multiple choice questions, please indicate which statement is correct by selecting an option (a), (b), (c) or (d). Only one option is correct.

Note: You get 4 points for each correct answer and -1 point for every incorrect answer.

Note: Please write your answers on your answer sheet (so **not** on this sheet). Clearly indicate your choice. No justifications are needed.

1. Consider the following Asymmetric GARCH model

$$x_t = \sigma_t \varepsilon_t \quad \text{for every} \quad t \in \mathbb{Z} \quad \text{where} \quad \{\varepsilon_t\}_{t\in\mathbb{Z}} \sim \text{NID}(0,1),$$

$$\text{where} \quad \sigma_t^2 = \omega + \alpha x_{t-1}^2 + \delta x_{t-1} + \beta \sigma_{t-1}^2 \quad \text{for every} \quad t \in \mathbb{Z}.$$

Which of the following statements is correct?

(a) A positive parameter $\delta > 0$ can be used to account for the *'Leverage Effect'*. If $\delta > 0$, then past negative returns $x_{t-1} < 0$ tend to have a smaller effect on the conditional volatility σ_t^2 than positive returns $x_{t-1} > 0$ of equal magnitude.

(b) A negative parameter $\delta < 0$ can be used to account for the *'Leverage Effect'*. If $\delta < 0$, then past negative returns $x_{t-1} < 0$ tend to have a smaller effect on the conditional volatility σ_t^2 than positive returns $x_{t-1} > 0$ of equal magnitude.

(c) A positive parameter $\delta > 0$ can be used to account for the *'Leverage Effect'*. If $\delta > 0$, then past negative returns $x_{t-1} < 0$ tend to have a larger effect on the conditional volatility σ_t^2 than positive returns $x_{t-1} > 0$ of equal magnitude.

(d) A negative parameter $\delta < 0$ can be used to account for the *'Leverage Effect'*. If $\delta < 0$, then past negative returns $x_{t-1} < 0$ tend to have a larger effect on the conditional volatility σ_t^2 than positive returns $x_{t-1} > 0$ of equal magnitude.

2. Let $\{x_t\}_{t\in\mathbb{Z}}$ be a strictly stationary and ergodic sequence with two bounded moments $\mathbb{E}|x_t|^2 < \infty$. Let $y_t = 5x_t^2$ and consider the sample average $\frac{1}{T}\sum_{t=1}^{T} y_t$.

Which of the following statements is correct?

(a) The sample average converges to the expectation $\mathbb{E}(x_t)$ by application of a law of large numbers.

(b) The sample average may or may not converge to the expectation $\mathbb{E}(x_t)$.

(c) The sample average converges to the expectation $\mathbb{E}(5x_t^2)$ by application of a law of large numbers.

(d) The sample average does not converge to the expectation $\mathbb{E}(5x_t^2)$.

3. Let $\{x_t\}_{t\in\mathbb{Z}}$ be a strictly stationary and ergodic sequence of stock returns with four bounded moments $\mathbb{E}|x_t|^4 < \infty$. Consider the following GARCH filtering equation for the conditional volatility with $\boldsymbol{\theta} := (\omega, \alpha, \beta) = (0.1, 0.15, 0.95)$,

$$\sigma_t^2 = 0.1 + 0.15x_{t-1}^2 + 0.95\sigma_{t-1}^2 \quad \text{for every} \quad t \in \mathbb{Z}.$$

Which of the following statements is correct?

(a) The filtered volatility $\{\hat{\sigma}_t^2(\boldsymbol{\theta}, \hat{\sigma}_1^2)\}_{t\in\mathbb{N}}$ is invertible at $\boldsymbol{\theta}$ and asymptotically stationary.

(b) The filtered volatility $\{\hat{\sigma}_t^2(\boldsymbol{\theta}, \hat{\sigma}_1^2)\}_{t\in\mathbb{N}}$ is not invertible at $\boldsymbol{\theta}$ and not asymptotically stationary.

(c) The filtered volatility $\{\hat{\sigma}_t^2(\boldsymbol{\theta}, \hat{\sigma}_1^2)\}_{t\in\mathbb{N}}$ is invertible at $\boldsymbol{\theta}$ but not asymptotically stationary.

(d) The filtered volatility $\{\hat{\sigma}_t^2(\boldsymbol{\theta}, \hat{\sigma}_1^2)\}_{t\in\mathbb{N}}$ is not invertible at $\boldsymbol{\theta}$ but it is asymptotically stationary.

4. Let θ_0 be the unique maximizer of the limit deterministic criterion $Q_\infty : \Theta \to \mathbb{R}$.

Which of the following statements is correct?

(a) θ_0 is identifiably unique.

(b) θ_0 may or may not be identifiably unique.

(c) θ_0 is identifiably unique if the parameter space Θ is compact.

(d) θ_0 is identifiably unique if the criterion converges uniformly.

5. Consider the following GARCH-in-mean model

$$x_t = \mu + \lambda\sigma_t + \sigma_t\varepsilon_t \quad \text{for every} \quad t \in \mathbb{Z} \quad \text{where} \quad \{\varepsilon_t\}_{t\in\mathbb{Z}} \sim \text{NID}(0, 1),$$

$$\text{where} \quad \sigma_t^2 = \omega + \alpha(x_{t-1} - \mu - \lambda\sigma_{t-1})^2 + \beta\sigma_{t-1}^2 \quad \text{for every} \quad t \in \mathbb{Z},$$

where $\omega \geq a > 0$, $\alpha > 0$ and $\beta > 0$. Say you want to estimate the parameter vector $\boldsymbol{\theta} = (\mu, \lambda, \omega, \alpha, \beta)$ using Maximum Likelihood (ML).

What is the correct expression of the log likelihood function that you will use for your estimation?

(a) $\sum_{t=2}^{T} -\frac{1}{2}\log(2\pi) - \frac{1}{2}\log(\hat{\sigma}_t^2(\boldsymbol{\theta}, \hat{\sigma}_1^2)) - \frac{(x_t-\mu)^2}{2\hat{\sigma}_t^2(\boldsymbol{\theta}, \hat{\sigma}_1^2)}$.

(b) $\sum_{t=2}^{T} -\frac{1}{2}\log(2\pi) - \frac{1}{2}\log(\hat{\sigma}_t^2(\boldsymbol{\theta}, \hat{\sigma}_1^2)) - \frac{(x_t-\mu-\lambda\hat{\sigma}_t^2(\boldsymbol{\theta}, \hat{\sigma}_1^2))^2}{2\hat{\sigma}_t^2(\boldsymbol{\theta}, \hat{\sigma}_1^2)}$.

(c) $\sum_{t=2}^{T} -\frac{1}{2}\log(2\pi) - \frac{1}{2}\log(\hat{\sigma}_t^2(\boldsymbol{\theta}, \hat{\sigma}_1^2)) - \frac{(x_t-\mu-\lambda\hat{\sigma}_t(\boldsymbol{\theta}, \hat{\sigma}_1^2))^2}{2\hat{\sigma}_t^2(\boldsymbol{\theta}, \hat{\sigma}_1^2)}$.

(d) None of the other answers are correct.

6. Consider the regression $y_t = f(x_t) + \varepsilon_t$.

   Select the correct statement:

   a. x_t is endogenous if x_t and ε_t are independent random variables.
   b. y_t is endogenous if y_t is independent of x_t.
   c. The problem of omitted variable bias can result in x_t being endogenous, in which case x_t is dependent on ε_t.
   d. The problem of simultaneity can lead to x_t being exogenous, in which case x_t is dependent on ε_t.

7. Let $\{x_t\}_{t\in\mathbb{Z}}$ be a sequence of independent random variables uniformly distributed on the interval $[-1, 1]$, i.e. $\{x_t\}_{t\in\mathbb{Z}} \sim \text{UID}([-1, 1])$.

   Which of the following statements is correct?

   (a) $\{x_t\}_{t\in\mathbb{Z}}$ is iid, white noise, weakly stationary, and strictly stationary.
   (b) $\{x_t\}_{t\in\mathbb{Z}}$ is iid, weakly stationary, and strictly stationary but not white noise.
   (c) $\{x_t\}_{t\in\mathbb{Z}}$ is strictly stationary but not white noise.
   (d) $\{x_t\}_{t\in\mathbb{Z}}$ is iid but not strictly stationary.

8. Let $\{x_t\}_{t\in\mathbb{Z}}$ be a strictly stationary and ergodic sequence with four bounded moments $\mathbb{E}|x_t|^4 < \infty$. Define $\boldsymbol{\theta} := (\omega, \alpha, \beta)$ and consider the following filtering equation for the conditional mean of $\{x_t\}_{t\in\mathbb{Z}}$,

$$\hat{\mu}_{t+1} = \omega + \alpha(x_t - \hat{\mu}_t) + \beta\hat{\mu}_t \quad \text{for every} \quad t \in \mathbb{N}.$$

   Which of the following statements is correct?

   (a) The filtered conditional mean $\{\hat{\mu}_t(\boldsymbol{\theta}, \hat{\mu}_1)\}_{t\in\mathbb{N}}$ is invertible at $\boldsymbol{\theta} \in \Theta$, if $|\alpha| < 1$.
   (b) The filtered conditional mean $\{\hat{\mu}_t(\boldsymbol{\theta}, \hat{\mu}_1)\}_{t\in\mathbb{N}}$ is invertible at $\boldsymbol{\theta} \in \Theta$, if $|\beta| < 1$, but not asymptotically stationary.
   (c) The filtered conditional mean $\{\hat{\mu}_t(\boldsymbol{\theta}, \hat{\mu}_1)\}_{t\in\mathbb{N}}$ is invertible at $\boldsymbol{\theta} \in \Theta$ and asymptotically stationary if $|\beta| < 1$.
   (d) The filtered conditional mean $\{\hat{\mu}_t(\boldsymbol{\theta}, \hat{\mu}_1)\}_{t\in\mathbb{N}}$ is invertible at $\boldsymbol{\theta} \in \Theta$ and asymptotically stationary if $|\beta - \alpha| < 1$.

9. Consider two competing models, A and B. Suppose you have at your disposal an estimation sample and an independent test sample.

   (a) Under appropriate regularity conditions, the *Hausman test* statistic can be used to test if model A predicts better than model B.
   (b) Under appropriate regularity conditions, the *Diebold-Mariano test* statistic can be used to test if model A nests model B.
   (c) Under appropriate regularity conditions, the *Hausman test* statistic can be used to test if model A is correctly specified using two different estimators for the parameters of model A.
   (d) Under appropriate regularity conditions, the *Diebold-Mariano test* statistic can be used to test if model A is correctly specified using two different estimators for the parameters of model A.

10. Consider the following random coefficient autoregressive model,

$$x_{t+1} = \beta_t x_t + \varepsilon_t$$

where $\{\beta_t\}_{t\in\mathbb{Z}}$ and $\{\varepsilon_t\}_{t\in\mathbb{Z}}$ are iid random variables with two bounded moments.

Which of the following statements is correct?

   (a) If $\mathbb{E}(\beta_t) < 1 \; \forall \; t$ then $\{x_t\}_{t\in\mathbb{Z}}$ is strictly stationary and ergodic.
   (b) $\mathbb{E}|\beta_t| < 1$ is a necessary condition for $\{x_t\}_{t\in\mathbb{Z}}$ to be stationary and ergodic.
   (c) $\mathbb{E}\log|\beta_t| < 1$ is a sufficient condition for $\{x_t\}_{t\in\mathbb{Z}}$ to be stationary and ergodic.
   (d) None of the other statements is correct.

Question 2 [30 points]
In economics and finance, time-series may sometimes exhibit time-varying conditional mean and volatility.

Let $\{x_t\}_{t\in\mathbb{Z}}$ be generated according to

$$x_t = \mu_t + \sigma_t \varepsilon_t \quad \text{for every} \quad t \in \mathbb{Z} \ ,$$

where $\{\varepsilon_t\}_{t\in\mathbb{Z}}$ is a sequence of Gaussian iid random variables $\{\varepsilon_t\}_{t\in\mathbb{Z}} \sim \text{NID}(0,1)$. Suppose that the time-varying conditional mean $\{\mu_t\}_{t\in\mathbb{Z}}$ satisfies

$$\mu_t = 0.2\big(x_{t-1} - \mu_{t-1}\big) + 0.7\mu_{t-1} \quad \text{for every} \quad t \in \mathbb{Z}.$$

Furthermore, let the time-varying volatility $\{\sigma_t\}_{t\in\mathbb{Z}}$ be determined by an exogenous sequence $\{z_t\}_{t\in\mathbb{Z}}$, according to

$$\sigma_t = \big(1 + \tanh(z_t)\big) \quad \text{for every} \quad t \in \mathbb{Z}.$$

Finally, let $\{z_t\}_{t\in\mathbb{Z}}$ be generated by the following random coefficient autoregressive model

$$z_{t+1} = \rho_t z_t + v_t \quad \text{for every} \quad t \in \mathbb{Z} \ ,$$

where $\{\rho_t\}_{t\in\mathbb{Z}}$ is a sequence of iid random variables with uniform distribution $\{\rho_t\}_{t\in\mathbb{Z}} \sim \text{UID}(0,\ 1.5)$ taking values in the interval $[0\ ,\ 1.5]$, and $\{v_t\}_{t\in\mathbb{Z}}$ is a sequence of Student-t iid random variables with two degrees of freedom $\{v_t\}_{t\in\mathbb{Z}} \sim TID(2)$.

Note: the acronym *iid* stands for *independent identically distributed.*
Note: the function $1 + \tanh(\cdot)$ is uniformly bounded between 0 and 2.
Note: the random variable v_t satisfies $\mathbb{E}|v_t|^n < \infty$ for $0 < n < 2$.

(a) **(13pts)** Can you show that $\{\sigma_t\}_{t\in\mathbb{Z}}$ is strictly stationary and ergodic?

(b) **(17pts)** Can you show that $\mathbb{E}|x_t|^2 < \infty$? Is $\{x_t\}_{t\in\mathbb{Z}}$ weakly stationary?

Question 3 [30 points]

Some econometricians claim that the temporal dependence in the growth rate of the *Gross Domestic Product* (GDP) is stronger during economic recession periods and weaker during expansions.

Let the sample of GDP growth rates $\{x_t\}_{t=1}^{T}$ at your disposal be a subset of the realized path of a strictly stationary and ergodic time-series $\{x_t\}_{t\in\mathbb{Z}}$ with bounded moments of fourth order $\mathbb{E}|x_t|^4 < \infty$. Consider the Gaussian logistic *Self Excited Smooth Transition Autoregressive* (SESTAR) model

$$x_t = \alpha + g(x_{t-1};\boldsymbol{\theta})x_{t-1} + \varepsilon_t \quad \text{for every} \quad t \in \mathbb{Z} \quad \text{where} \quad \{\varepsilon_t\}_{t\in\mathbb{Z}} \sim \text{NID}(0,\sigma_\varepsilon^2)$$

$$\text{and} \quad g(x_{t-1};\boldsymbol{\theta}) := \frac{\gamma}{1+\exp(\beta x_{t-1})} \quad \text{for every} \quad t \in \mathbb{Z}.$$

Suppose that the parameters $\boldsymbol{\theta} = (\alpha, \gamma, \beta, \sigma_\varepsilon^2)$ of the model are estimated by maximum likelihood (ML) on a compact parameter space Θ with $\sigma_\varepsilon^2 > 0$. Note also that $g(x;\boldsymbol{\theta})$ is uniformly bounded since $|g(x;\boldsymbol{\theta})| \leq |\gamma|$ for every $(x,\boldsymbol{\theta})$. Suppose that the ML estimator $\hat{\boldsymbol{\theta}}_T$ is consistent for a parameter $\boldsymbol{\theta}_0$ in the interior of Θ.

(a) **(13pts)** Can you derive the limit distribution of the derivative of the log likelihood function evaluated at $\boldsymbol{\theta}_0$ (multiplied by $\sqrt{T}$)?

(b) **(10pts)** Explain how you would show asymptotic normality of $\hat{\boldsymbol{\theta}}_T$. Be explicit about the conditions and concepts you would use. No derivations are needed.
Note: you can assume that certain functions are well behaved and continuously differentiable.

(c) **(7pts)** Explain how you can use the approximate distribution of $\hat{\boldsymbol{\theta}}_T$ to test the claim that the temporal dependence in the growth rate of Dutch GDP is stronger during economic recession periods and weaker during expansions.