vrije Universiteit amsterdam

Preparation Exam
Advanced Econometrics (4.1)

Master Econometrics and Operations Research
School of Business and Economics

Exam:	Advanced Econometrics (4.1)
Code:	E_EORM_AECTR
Coordinator:	F. Blasques
Date:	–
Time:	–
Duration:	2 hours and 45 minutes
Calculator:	Not allowed
Graphical calculator:	Not allowed
Number of questions:	3
Type of questions:	Open
Answer in:	English
Credit score:	100 credits counts for a 10
Grades:	Made public within 10 working days
Inspection:	By appointment (send e-mail to f.blasques@vu.nl)
Number of pages:	6, including front page

- Read the entire exam carefully before you start answering the questions.
- Be clear and concise in your statements, but justify every step in your derivations.
- If you think that further information is needed to answer a question or that the question is ill-posed, then explain your reasoning.
- The questions should be handed back at the end of the exam. Do not take it home.

Good luck!

Question 1 [28 points]   Multiple Choice

For each of the following multiple choice questions, please indicate which statement is correct by selecting option (a), (b), (c), or (d), or alternatively, if none of the options are correct, please state 'No answer is correct'.

All answers should be written on the answer sheet clearly and in order.

Note: You get 4 points for each correct answer and -1 point for every incorrect answer.

Note: If you believe that more than one statement is correct, then please indicate which answers are correct and why.

Note: Please write your answers in the answer sheet. Clearly indicate your choice. No justifications are needed.

1. Consider the following Asymmetric GARCH model

$$x_t = \sigma_t \varepsilon_t \quad \text{for every} \quad t \in \mathbb{Z} \quad \text{where} \quad \{\varepsilon_t\}_{t\in\mathbb{Z}} \sim \text{NID}(0,1),$$
$$\text{where} \quad \sigma_t^2 = \omega + \alpha x_{t-1}^2 + \delta x_{t-1} + \beta \sigma_{t-1}^2 \quad \text{for every} \quad t \in \mathbb{Z}.$$

   (a) A positive parameter $\delta > 0$ can be used to account for the *'Leverage Effect'*. If $\delta > 0$, then past negative returns $x_{t-1} < 0$ tend to have a smaller effect on the conditional volatility σ_t^2 than positive returns $x_{t-1} > 0$ of equal magnitude.

   (b) A negative parameter $\delta < 0$ can be used to account for the *'Leverage Effect'*. If $\delta < 0$, then past negative returns $x_{t-1} < 0$ tend to have a smaller effect on the conditional volatility σ_t^2 than positive returns $x_{t-1} > 0$ of equal magnitude.

   (c) A positive parameter $\delta > 0$ can be used to account for the *'Leverage Effect'*. If $\delta > 0$, then past negative returns $x_{t-1} < 0$ tend to have a larger effect on the conditional volatility σ_t^2 than positive returns $x_{t-1} > 0$ of equal magnitude.

   (d) A negative parameter $\delta < 0$ can be used to account for the *'Leverage Effect'*. If $\delta < 0$, then past negative returns $x_{t-1} < 0$ tend to have a larger effect on the conditional volatility σ_t^2 than positive returns $x_{t-1} > 0$ of equal magnitude.

2. Let $\{x_t\}_{t\in\mathbb{Z}}$ be a strictly stationary and ergodic sequence with two bounded moments $\mathbb{E}|x_t|^2 < \infty$. Let $y_t = 5x_t^2$. Consider the sample average $\frac{1}{T}\sum_{t=1}^{T} y_t$.

   (a) The sample average converges to the expectation $\mathbb{E}(x_t)$ by application of a law of large numbers.

   (b) The sample average may or may not converge to the expectation $\mathbb{E}(x_t)$.

   (c) The sample average converges to the expectation $\mathbb{E}(5x_t^2)$ by application of a law of large numbers.

   (d) The sample average does not converge to the expectation $\mathbb{E}(5x_t^2)$.

Question 1 [28 points] - Multiple Choice (continued)

3. Consider the following regression model,

$$y_t = \alpha + \beta x_t + \varepsilon_t.$$

Suppose that you wish to perform structural econometric analysis, but the regressor x_t is endogenous, i.e. $\mathbb{E}(\varepsilon_t|x_t) \neq 0$. For this reason, you consider using another variable z_t as an instrument in the following regression

$$x_t = \delta + \gamma z_t + v_t.$$

Luckily, you find that the variable z_t is highly correlated with x_t, so it useful in predicting the endogenous regressor x_t.

(a) If z_t is independent of ε_t, then z_t is a *valid instrument.*

(b) If z_t is independent of both ε_t and v_t, then z_t is a *valid instrument.*

(c) If z_t is independent of v_t, then z_t is a *valid instrument.*

(d) If z_t is independent of both ε_t and y_t, then z_t is. a *valid instrument.*

4. Let $\{x_t\}_{t\in\mathbb{Z}}$ be a strictly stationary and ergodic sequence of stock returns with four bounded moments $\mathbb{E}|x_t|^4 < \infty$. Consider the following GARCH filtering equation for a the conditional volatility with $\boldsymbol{\theta} := (\omega, \alpha, \beta) = (0.1, 0.15, 0.95)$,

$$\sigma_t^2 = 0.1 + 0.15x_{t-1}^2 + 0.95\sigma_{t-1}^2 \quad \text{for every} \quad t \in \mathbb{Z}.$$

(a) The filtered volatility $\{\hat{\sigma}_t^2(\boldsymbol{\theta}, \hat{\sigma}_1^2)\}_{t\in\mathbb{N}}$ is invertible at $\boldsymbol{\theta}$ and asymptotically stationary.

(b) The filtered volatility $\{\hat{\sigma}_t^2(\boldsymbol{\theta}, \hat{\sigma}_1^2)\}_{t\in\mathbb{N}}$ is not invertible at $\boldsymbol{\theta}$ and not asymptotically stationary.

(c) The filtered volatility $\{\hat{\sigma}_t^2(\boldsymbol{\theta}, \hat{\sigma}_1^2)\}_{t\in\mathbb{N}}$ is invertible at $\boldsymbol{\theta}$ but not asymptotically stationary.

(d) The filtered volatility $\{\hat{\sigma}_t^2(\boldsymbol{\theta}, \hat{\sigma}_1^2)\}_{t\in\mathbb{N}}$ is not invertible at $\boldsymbol{\theta}$ but it is asymptotically stationary.

Question 1 [28 points] - Multiple Choice (continued)

5. Consider the random sample $\{x_t\}_{t=1}^T$. Consider the extremum estimator

$$\hat{\theta}_T \in \arg\max_{\theta\in\Theta} \frac{1}{T}\sum_{t=1}^{T}(x_t - \theta) \quad \text{where } \Theta = [2, 10].$$

   (a) $\hat{\theta}_T$ is not consistent.
   (b) $\hat{\theta}_T \xrightarrow{p} 10$ as $T \to \infty$.
   (c) $\hat{\theta}_T = 2$ almost surely.
   (d) $\hat{\theta}_T = 10$ almost surely.

6. Consider the M-estimator $\hat{\theta}_T$ given by $\hat{\theta}_T \in \arg\max_{\theta\in\Theta} Q_T(\theta)$. Suppose that the estimator is consistent for $\theta_0 \in \Theta$, i.e. $\hat{\theta}_T \xrightarrow{p} \theta_0$ as $T \to \infty$.

   (a) $\hat{\theta}_T$ is asymptotically normal if $\sqrt{T}\hat{\theta}_T \xrightarrow{d} N(0, V)$, as $T \to \infty$, where V denotes the asymptotic variance of the estimator.
   (b) $\hat{\theta}_T$ is asymptotically normal if $\sqrt{T}\hat{\theta}_T \xrightarrow{d} N(\theta_0, V)$, as $T \to \infty$, where V denotes the asymptotic variance of the estimator.
   (c) $\hat{\theta}_T$ is asymptotically normal if $\sqrt{T}(\hat{\theta}_T - \theta_0) \xrightarrow{d} N(\theta_0, V)$, as $T \to \infty$, where V denotes the asymptotic variance of the estimator.
   (d) $\hat{\theta}_T$ is asymptotically normal if $\sqrt{T}\hat{\theta}_T/V \xrightarrow{d} N(0, 1)$, as $T \to \infty$, where V denotes the asymptotic variance of the estimator.

7. Let $\mathbf{x}_T := (x_1, ..., x_T)$ be a subset of a strictly stationary and ergodic sequence $\{x_t\}_{t\in\mathbb{Z}}$ satisfying $\mathbb{E}|x_t|^4 < \infty$. Consider the AR(1) model,

$$x_t = \theta x_{t-1} + \varepsilon_t \quad \text{for every} \quad t \in \mathbb{Z}.$$

   You have decided to use the following estimator for the true parameter θ_0,

$$\hat{\theta}_T \in \arg\max_{\theta\in\Theta} -\frac{1}{T}\sum_{t=2}^{T}\left[\frac{u_t(\theta)^2}{1 + u_t(\theta)^2} + \big(\theta - 0.9\big)^2\right],$$

   where $u_t(\theta) = x_t - \theta x_{t-1}$

   (a) $\hat{\theta}_T$ cannot be consistent for θ_0.
   (b) $\hat{\theta}_T$ can only be consistent if $\theta_0 = 0.9$.
   (c) If the least squares estimator $\tilde{\theta}_T \in \arg\max_{\theta\in\Theta} -\frac{1}{T}\sum_{t=2}^{T} u_t(\theta)^2$ is consistent, then $\hat{\theta}_T$ is also consistent.
   (d) $\hat{\theta}_T$ is always consistent for θ_0.

Question 2 [35 points] Stochastic Properties of Nonlinear Dynamic Models

In economics and finance, time-series may sometimes exhibit time-varying conditional mean and volatility.

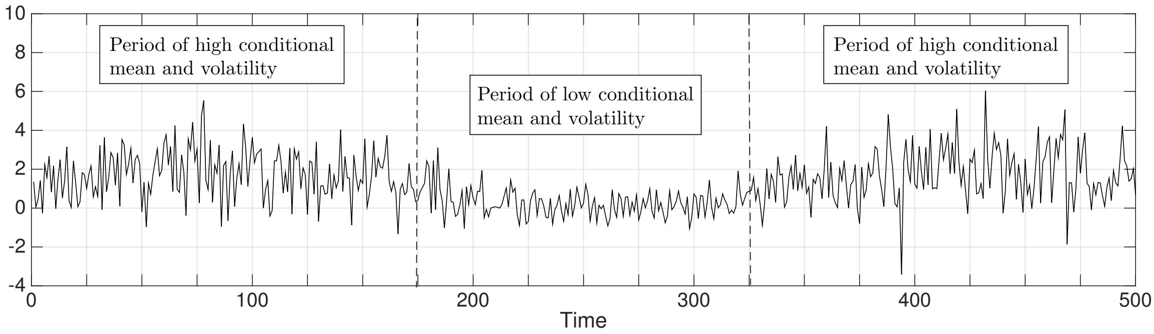

Figure 1: Time-series with time-varying conditional mean and volatility.

Let $\{x_t\}_{t\in\mathbb{Z}}$ be generated according to

$$x_t = \mu_t + \sigma_t \varepsilon_t \quad \text{for every} \quad t \in \mathbb{Z} \ ,$$

where $\{\varepsilon_t\}_{t\in\mathbb{Z}}$ is a sequence of Gaussian iid random variables $\{\varepsilon_t\}_{t\in\mathbb{Z}} \sim \text{NID}(0, 1)$. Suppose that the time-varying conditional mean $\{\mu_t\}_{t\in\mathbb{Z}}$ satisfies

$$\mu_t = 0.2\big(x_{t-1} - \mu_{t-1}\big) + 0.7\mu_{t-1} \quad \text{for every} \quad t \in \mathbb{Z}.$$

Furthermore, let the time-varying volatility $\{\sigma_t\}_{t\in\mathbb{Z}}$ be determined by an exogenous sequence $\{z_t\}_{t\in\mathbb{Z}}$, according to

$$\sigma_t = \big(1 + \tanh(z_t)\big) \quad \text{for every} \quad t \in \mathbb{Z}.$$

Finally, let $\{z_t\}_{t\in\mathbb{Z}}$ be generated by the following random coefficient autoregressive model

$$z_{t+1} = \rho_t z_t + v_t \quad \text{for every} \quad t \in \mathbb{Z} \ ,$$

where $\{\rho_t\}_{t\in\mathbb{Z}}$ is a sequence of iid random variables with uniform distribution $\{\rho_t\}_{t\in\mathbb{Z}} \sim \text{UID}(0,\ 1.5)$ taking values in the interval $[0\ ,\ 1.5]$, and $\{v_t\}_{t\in\mathbb{Z}}$ is a sequence of Student-t iid random variables with two degrees of freedom $\{v_t\}_{t\in\mathbb{Z}} \sim TID(2)$.

Note: the acronym *iid* stands for *independent identically distributed.*
Note: the function $1 + \tanh(\cdot)$ is uniformly bounded between 0 and 2.
Note: the random variable v_t satisfies $\mathbb{E}|v_t|^n < \infty$ for $0 < n < 2$.

(a) Can you show that $\{\sigma_t\}_{t\in\mathbb{Z}}$ is strictly stationary and ergodic?

(b) Can you show that $\mathbb{E}|x_t|^2 < \infty$? Is $\{x_t\}_{t\in\mathbb{Z}}$ weakly stationary?

Question 3 [37 pts] - Estimation and Inference for Nonlinear Dynamic Models

Some econometricians claim that the temporal dependence in the growth rate of the *Gross Domestic Product* (GDP) is stronger during economic recession periods and weaker during expansions. Figure 1 plots the *growth rate of quarterly real GDP in The Netherlands.*

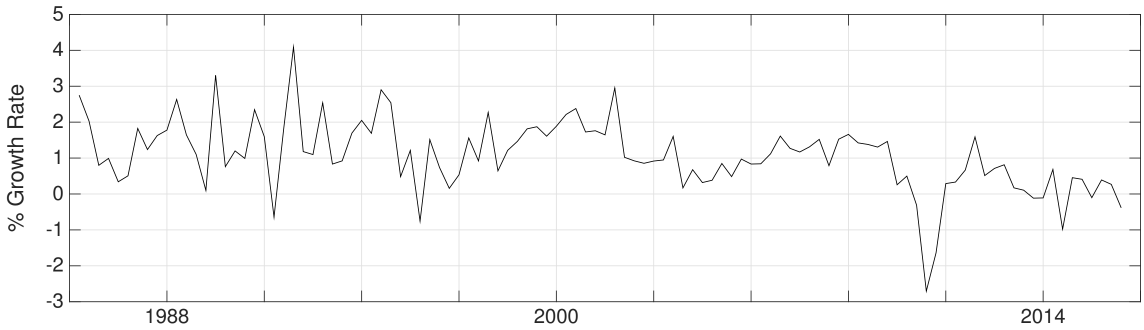

Figure 2: Real GDP growth rate for The Netherlands (in percentage).

Let the sample of GDP growth rates $\{x_t\}_{t=1}^{T}$ at your disposal be a subset of the realized path of a strictly stationary and ergodic time-series $\{x_t\}_{t\in\mathbb{Z}}$ with bounded moments of fourth order $\mathbb{E}|x_t|^4 < \infty$. Consider the Gaussian logistic *Self Excited Smooth Transition Autoregressive* (SESTAR) model

$$x_t = \alpha + g(x_{t-1};\boldsymbol{\theta})x_{t-1} + \varepsilon_t \quad \text{for every} \quad t \in \mathbb{Z} \quad \text{where} \quad \{\varepsilon_t\}_{t\in\mathbb{Z}} \sim \text{NID}(0,\sigma_\varepsilon^2)$$

$$\text{and} \quad g(x_{t-1};\boldsymbol{\theta}) := \frac{\gamma}{1+\exp(\beta x_{t-1})} \quad \text{for every} \quad t \in \mathbb{Z}.$$

Suppose that the parameters $\boldsymbol{\theta} = (\alpha,\gamma,\beta,\sigma_\varepsilon^2)$ of the model are estimated by maximum likelihood (ML) on a compact parameter space Θ with $\sigma_\varepsilon^2 > 0$. Note also that $g(x;\boldsymbol{\theta})$ is uniformly bounded since $|g(x;\boldsymbol{\theta})| \leq |\gamma|$ for every $(x,\boldsymbol{\theta})$.

(a) Suppose that the ML estimator $\hat{\boldsymbol{\theta}}_T$ is consistent for a parameter $\boldsymbol{\theta}_0$ in the interior of Θ. Can you obtain an approximate distribution for the ML estimator?
Note: you can assume that certain functions are well behaved and continuously differentiable.

(b) Explain how you can use the approximate distribution of $\hat{\boldsymbol{\theta}}_T$ to test the claim that the temporal dependence in the growth rate of Dutch GDP is stronger during economic recession periods and weaker during expansions.