vrije Universiteit *amsterdam*

Faculty of Economics and Business Administration

Exam:	Advanced Econometrics (part 2) (4.2)
Code:	FEWEB_E_EORM_AECTR_2015_111
Examinator:	Ryo Okui / Katarzyna Lasak
Co-reader:	Katarzyna Lasak / Ryo Okui
Date:	December 14, 2015
Time:	18:30
Duration:	2 hours
Calculator allowed:	Yes
Graphical calculator allowed:	Yes
Number of questions:	3
Type of questions:	Open
Answer in:	English

Remarks: Students can bring one A4 paper whose contents can be anything. Both side of the paper can be used. It must be one piece of paper (it is not allowed to bring two A5 papers).

Credit score:	100 credits in total. 35 points for the 1st question, 25 points for the 2nd question and 40 points for the 3rd question
Grades:	The grades will be made public on: January 11, 2016
Inspection:	Tuesday, January 19th at 15:00 in room 10A-28.
Number of pages:	3 (including front page)

Good luck!

. (35 points)

We consider the following VAR(1) model:

$$\mathbf{y}_t = \mathbf{c} + \mathbf{A}_1 \mathbf{y}_{t-1} + \mathbf{u}_t,$$

where $\mathbf{y}_t$ is a 2×1 vector of random variables, $\mathbf{c}$ is a 2×1 vector of deterministic elements, $\mathbf{A}_1$ is a 2×2 matrix with deterministic elements and $\mathbf{u}_t \sim i.i.d.N(0, \Sigma)$ and Σ is a 2×2 matrix with deterministic elements.

Suppose that

$$\mathbf{c} = \begin{pmatrix} 1 \\ 2 \end{pmatrix},$$

$$\mathbf{A}_1 = \begin{pmatrix} 0.6 & 0.7 \\ 0.1 & 0.2 \end{pmatrix} \quad y_{t-1} \; y_{t-2}$$

and

$$\Sigma = \begin{pmatrix} 9 & 3 \\ 3 & 5 \end{pmatrix}$$

(a) Is $\mathbf{y}_t$ stable?

(b) Derive $E(\mathbf{y}_t)$.

(c) Let $\mathbf{y}_t = (y_{1t}, y_{2t})'$. Does y_{1t} Granger–cause y_{2t}? Does y_{2t} Granger–cause y_{1t}?

(d) Compute the (non-orthogonal) impulse response function of y_{1t} with respect to the second error for $h = 0, 1, 2$. We consider a one standard deviation shock.

(e) Compute the orthogonal impulse response function of y_{2t} with respect to the first error for $h = 0, 1, 2$. We use the Cholesky decomposition and we consider a one standard deviation shock.

(f) Compute the relative variance contribution of y_{1t} with respect to the second error for $h = 1, 2$. We use the Cholesky decomposition.

(g) We have learned in the class that there are two ways to obtain confidence intervals for impulse response functions and relative variance contributions: Delta method and bootstrap. For each of these method, explain how we obtain confidence intervals in at most 10 lines.

(h) Explain why nonparametric bootstrap does not work for obtaining confidence intervals of impulse response functions and relative variance contributions.

. (25 points)

Suppose that a univariate time series $\{y_t\}$ is generated by:

$$y_t = y_{t-1} + u_t,$$

where u_t follows a stationary AR(1) process such that $u_t = \alpha u_{t-1} + \epsilon_t$, $\epsilon_t \sim N(0, \sigma^2)$ and $|\alpha| < 1$, and y_0 is given.

(a) Derive the long-run variance of u_t.

(b) Suppose that you know that u_t is generated by a stationary AR(1) process but you do not know the values of α and σ^2. Explain how to estimate the long-run variance of u_t based on the estimates of α and σ^2. Eerst met OLS de α berekenen en $\hat{\sigma}^2 = \ldots$

(c) Suppose that you do not know the data generating process of u_t. Explain how to estimate the long-run variance using a kernel function in at most 5 lines.

(d) Suppose that we estimate the following AR model:

$$y_t = \rho y_{t-1} + v_t.$$

Derive the asymptotic distribution of the t-test statistics for the null hypothesis that $\rho = 1$.

(e) Let $\hat{\lambda}^2$ be a consistent estimate of the long-run variance of u_t. Let $\hat{\sigma}^2$ be a consistent estimate of σ^2.
Write down the formula of the Phillips-Perron test statistics for the null hypothesis that $y_t = y_{t-1} + u_t$ and the alternative that $y_t = \rho y_{t-1} + u_t$ with $|\rho| < 1$.
Derive the asymptotic distribution of the Phillips-Perron test statistics.

3. (40 points) (Please write your answer to this question on a separate sheet of paper)

(a) Define a long memory process and describe its most important characteristic. Describe one of the mechanisms that generates long memory in economic models.

(b) Define (fractional) cointegration. Describe possible pitfalls of using standard cointegration framework to analyse fractionally cointegrated series. What do fractional cointegration and standard cointegration concepts have in common?

(c) Consider fractionally cointegrated VECM model

$$\Delta^d X_t = \alpha\beta'(\Delta^{d-b} - \Delta^d)X_t + \Phi D_t + \varepsilon_t, \qquad (1)$$

where X_t is an $I(d)$ vector of dimension $p \times 1$, b fractional cointegration degree, α adjustment coefficient matrix, β cointegrating vectors matrix, ε_t vector of IID Gaussian errors with covariance matrix Ω, Φ is a matrix of freely varying parameters and D_t is a matrix with deterministic terms and other exogenous variables.

Describe (in steps) the estimation procedure (for all the parameters) based on reduced rank regressions for the model (1). What are the properties of the estimators?

(d) Derive the following tests:

- maximum eigenvalue test for testing the null hypothesis of no cointegration in model (1).

- trace test for testing the null hypothesis of cointegration rank $r = 1$ in model (1).

What are the properties of these tests (with respect to the standard ones)?

(e) Compare the properties of the following two fractional models with short run dynamics:

$$\Delta^d X_t = \alpha\beta'(\Delta^{d-b} - \Delta^d)X_t + \sum_{j=1}^{k} \Gamma_j \Delta^d L_b^j X_t + \varepsilon_t,$$

and

$$\Delta^d X_t = \alpha\beta'(\Delta^{d-b} - \Delta^d)X_t + \sum_{j=1}^{k} L^j B_j \left\{(\Delta^{d-b} - \Delta^d)X_t\right\} + \sum_{j=1}^{k} L^j A_j \Delta^d X_t + \varepsilon_t,$$

(f) Give an example of an empirical problem that could be analysed in (fractional) cointegration framework. Explain why you expect cointegration to be of fractional order.

(g) What are the possible gains of introducing long memory into models of volatility? Give an example of such a model.

THANK YOU!