advanced Econometrics deel I

Question 1 (50 out of 100 points)

Consider the following n-dimensional vector autoregressive (VAR) process of order 2 for $t = 3, \ldots, T$:

$$y_t = \gamma + \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \varepsilon_t \quad (1)$$

$$\varepsilon_t \sim \mathsf{NID}(0, \Sigma). \quad (2)$$

The values of y_1 and y_2 are known.

1. Provide two equivalent formulations of the conditions under which $\{y_t\}$ in (1)-(2) is weakly stationary. *NB: The terms 'weak stationarity' and 'covariance stationarity' are synonymous.*

2. Derive the log-likelihood function of the sample conditional on y_1 and y_2. You may use the fact that the density of a multivariate Normal random variable $z \sim \mathsf{N}(\mu, \Gamma)$ is

$$f(z) = (\det(2\pi\Gamma))^{-1/2} \exp\left(-\frac{1}{2}(z-\mu)'\Gamma^{-1}(z-\mu)\right).$$

Assume for simplicity that in (1)-(2) the dimension $n = 2$ such that, for instance, y_t is the (2×1) vector

$$y_t = \begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix}.$$

3. Define the concept of Granger-causality, and explain how you would test the null hypothesis that y_{2t} does not Granger-cause y_{1t} in the model (1)-(2).

Consider now the full-sample representation of a seemingly unrelated regressions (SUR) model

$$y = X\beta + \varepsilon \quad (3)$$

$$\varepsilon \sim \mathsf{N}(0, \Omega) \quad (4)$$

where

$$\beta = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_n \end{pmatrix}, \quad X = \begin{pmatrix} X_1 & & 0 \\ & \ddots & \\ 0 & & X_n \end{pmatrix}, \quad y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_n \end{pmatrix}$$

with $X_i = (x_{i1} : \cdots : x_{iT})'$, $y_i = (y_{i1} : \cdots : y_{iT})'$ and similarly for ε_i, $i = 1, \ldots, n$. While y_{it} and ε_{it} are scalars both x_{it} and β_i are vectors of dimension $(k \times 1)$. Importantly, it is assumed that

$$\mathsf{E}(\varepsilon_{it}) = 0$$

$$\mathsf{E}(\varepsilon_{it}\varepsilon_{js}) = \begin{cases} \sigma_{ij}, & \text{when } t = s \\ 0, & \text{when } t \neq s. \end{cases}$$

4. Argue that the GLS estimator of β in (3), i.e.

$$\hat{\beta} = \left(X'\Omega^{-1}X\right)^{-1} X'\Omega^{-1}y,$$

has minimum variance amongst all linear unbiased estimators of β in model (3)-(4).

5. Show that the VAR(2) model in (1)-(2) may be written in SUR form (3)-(4) and that it may in fact be efficiently estimated by OLS.

Question 2 (50 out of 100 points)

Consider an autoregressive distributed lag (ADL) model of order $(1, 1)$:

$$y_t = \alpha + \phi_1 y_{t-1} + \beta_0 x_t + \beta_1 x_{t-1} + \varepsilon_t \quad (5)$$

$$\varepsilon_t \sim \mathsf{NID}(0, \sigma^2) \quad (6)$$

1. Derive the error correction representation of (5)-(6). Interpret the components of the error correction form.

Interest now lies on a stochastic process $\{x_t\}$ and one suspects that it is a random walk.

2. Is a random walk mean-reverting? Are its shocks transient? Justify your answers.

Suppose that the relationship between two stochastic processes $\{x_t\}$ and $\{y_t\}$ is investigated, both of which are known to be integrated of order 1, i.e. they are $\mathsf{I}(1)$.

3. Provide intuition behind the concept of cointegration.

4. Explain in detail how you would test for cointegration between x_t and y_t using the Engle-Granger 2-step procedure.

The following Monte Carlo experiment was conducted: Data of the processes

$$x_t = 0.8x_{t-1} + u_t$$

$$y_t = 0.8y_{t-1} + v_t$$

$$\begin{pmatrix} u_t \\ v_t \end{pmatrix} \sim \mathsf{NID}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]$$

were generated for $t = 1, \ldots, T$. Subsequently, the model

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t, \quad \varepsilon_t \sim \mathsf{NID}(0, \sigma^2)$$

was estimated by OLS and a t-test for the hypothesis

$$H_0 : \beta_1 = 0 \quad (7)$$

was conducted against the alternative $H_1 : \beta_1 \neq 0$ at the 5% level. In one setting, the t statistic was computed using OLS standard errors; in a second setting, heteroscedasticity and autocorrelation consistent (HAC) standard errors were used. The sample sizes T that were considered are 50, 100, 200, 400, 800, 1600, 3200, 6400. The number of replications in the Monte Carlo experiment was 10000.

T	50	100	200	400	800	1600	3200	6400
t statistic w/ OLS SEs	33.94%	34.92%	35.08%	35.75%	35.87%	36.45%	36.50%	35.65%
t statistic w/ HAC SEs	29.74%	20.80%	13.33%	10.36%	7.85%	6.64%	6.31%	5.17%

Table 1: empirical rejection frequencies of the null hypothesis in (7).

5. Table 1 displays the empirical rejection frequencies of H_0 for both varieties of the t statistic and all sample sizes. Explain these findings.