vrije Universiteit *amsterdam*

Faculty of Economics and Business Administration

Exam:	Adv Econometrics I 4.1
Code:	E_EORM_AECTR
Coordinator:	K. A. Lasak
Date:	October 22, 2012
Time:	08:45h
Duration:	2 hours 45 minutes 11.30
Calculator allowed:	No
Graphical calculator allowed:	No
Number of questions:	3
Type of questions:	Open
Answer in:	English

Remarks:
Exam will be handed to attendants in room HG-04A05.

Credit score:	100 credits counts for a 10
Grades:	The grades will be made public on Monday, November 5, 2012
Inspection:	By appointment
Number of pages:	4 (including this page)

Good luck!

EXAM Advanced Econometrics 4.1

22 October 2012, 08.45 – 11.30 a.m.

- This exam consists of a total of 4 numbered pages.
- You have 2 hours 45 minutes for the exam.
- Read the entire question, before starting to answer. Questions are not explicitly in order of difficulty, so temporarily skip sub-questions if you do not know the answer.
- **Motivate all your answers** and computations.
- Describe your derivations clearly. Use clear notation.
- Be **concise** in all your answers.
- Answers should be in English.
- For each sub-question the number of points (on a scale of 100) is clearly indicated.
- The questions should be handed back at the end of the exam. **You may not take them with you!**

Good luck!

1. (35/100) Consider the linear regression with the error term following an AR(1) process, i.e.:

$$y_t = X_t\beta + u_t \qquad u_t = \rho u_{t-1} + \epsilon_t \qquad \epsilon_t \sim iid(0, \sigma_\epsilon^2) \qquad |\rho| < 1 \qquad t = 1, \ldots, n. \qquad (1)$$

(a) (6/100) Derive a dynamic version of this model and discuss how to test for AR(1) effects.

(b) (6/100) Discuss how to test whether u_t is homoskedastic or heteroskedastic.

(c) (6/100) Derive asymptotic distribution of β_{MM}, Method of Moments estimator of β. Indicate all assumptions you need to derive your result.

(d) (6/100) Explain how to choose weighting matrix (W) to get efficient β_{MM}.

(e) (5/100) Write down explicitly the form of $\Omega(\rho)$, the covariance matrix of the vector u.

(f) (6/100) Describe efficient estimation of β based on the decomposition $\Omega^{-1} = \Psi\Psi^T$.

2. (35/100) Consider the following linear regression model:

$$y = X\beta + u \qquad E(uu') = \sigma^2 I, \qquad (2)$$

where at least one of the explanatory variables in the $n \times k$ matrix X is assumed not to be predetermined with respect to the error term u. Suppose for each observation t we can find $E(u_t|\Omega_t) = 0$ and we can form $n \times l$ matrix W s.t. $W_t \in \Omega_t$, where u_t and W_t denote $t-$th element of u and W respectively. Ω_t is an information set in period t, $t = 1, \ldots, n$.

(a) (5/100) Show that OLS is no longer consistent in this case.

(b) (6/100) State the criterion function that you minimize to obtain an efficient estimator of β. Demonstrate that F.O.C for minimization are equivalent to the moment conditions. Derive resulting estimator $\hat{\beta}$.

(c) (6/100) Discuss consistency of $\hat{\beta}$.

(d) (6/100) Derive the asymptotic distribution of $\sqrt{n}(\hat{\beta} - \beta_0)$, where β_0 is true value of β.

(e) (6/100) Discuss the optimal choice of instruments (W) in just identified case $l = k$.

(f) (6/100) Discuss the optimal choice (in terms of J, an $l \times k$ matrix) of effective instruments (WJ) in overidentified case. i.e. when $l > k$.

3. (30/100) Consider GMM estimation with the columns of the $n \times l$ matrix W used as instrumental variables (n is number of observations, l is number of instruments). Suppose also the $n-$vector $f(\theta)$ of elementary zero functions has a covariance matrix $\sigma^2 I$ (except in question 3(c)).

   (a) (6/100) Show that the GMM criterion function is

$$\frac{1}{\sigma^2} f^T(\theta) P_W \, f(\theta). \tag{3}$$

   where f^T stands for transpose of f and P_W is orthogonal projection on to the space of instrumental variables W.

   (b) Show that, whenever the instruments are predetermined, the artificial regression

$$f(\theta) = -P_W F(\theta) b + residuals, \tag{4}$$

   where b is regression parameter, $n \times k$ matrix $F(\theta)$ has typical element $F_{ti}(\theta) \equiv \frac{\partial f_t(\theta)}{\partial \theta_i}$,with θ_i the $i-th$ element of θ, $i = 1, \ldots k$, $t = 1, \ldots n$, satisfies all the requisite properties for hypothesis testing. i.e.

      i. (6/100) the regressand is orthogonal to the regressors when they are evaluated at the GMM estimator $\hat{\theta}$ obtained by minimizing (3);

      ii. (6/100) the OLS covariance matrix from (4) is a consistent estimate of the asymptotic variance of that estimator;

      iii. (6/100) regression (4) admits one-step estimator. i.e. when (4) is evaluated at any consistent estimator $\acute{\theta}$, the OLS parameter estimates $\acute{b}$ are such that $n^{1/2}(\hat{\theta} - \acute{\theta}) \stackrel{a}{=} n^{1/2}\acute{b}$.

   (c) (6/100) Derive a heteroskedasticity robust version of the artificial regression (4), assuming that the covariance matrix of the vector $f(\theta)$ of elementary zero functions is diagonal, but otherwise arbitrary.