1. (25/100) Consider the linear regression with the error term following an AR(1) process, i.e.:

$$y_t = X_t\beta + u_t \qquad u_t = \rho u_{t-1} + \epsilon_t \qquad \epsilon_t \sim iid(0, \sigma_\epsilon^2) \qquad |\rho| < 1 \qquad t = 1, \ldots, n. \quad (1)$$

   (a) (2/100) Derive a dynamic version of this model and name the method that estimates β efficiently in this case.

   (b) (2/100) Write down the model derived in (a) as a nonlinear model in a matrix notation. Write down the moment conditions for estimating β using instruments W. How do you interpret moment conditions geometrically?

   (c) (1/100) Let $\hat{\beta}$ be the resulting method of moments estimator of β. State necessary and sufficient conditions for consistency of $\hat{\beta}$.

   (d) (15/100) Derive the asymptotic distribution of $\sqrt{n}(\hat{\beta} - \beta_0)$. Indicate all assumptions you need to make to derive your result.

   (e) (5/100) Find optimal set of instruments W and demonstrate that this choice is indeed optimal. Discuss whether optimal instruments are feasible. In case they are not suggest how you would estimate this model efficiently. Did you get the same answer as in point (a.)?

2. (25/100) Consider the following linear regression model:

$$y = X\beta + u \qquad E(uu') = \sigma^2 I, \quad (2)$$

   where at least one of the explanatory variables in the $n \times k$ matrix X is assumed not to be predetermined with respect to the error term u.

   (a) (1/100) Which undesired properties $\hat{\beta}_{OLS}$ has in this case?

   (b) (2/100) Give two examples of common situations, in which X is not predetermined with respect to the error term.

   (c) (10/100) Consider IV estimation of the model (2) with instruments $P_W X$. Write down IV-GNR and discuss its properties relative to the model (2).

   (d) (6/100) Discuss testing linear restrictions based on IV-GNR.

   (e) (6/100) Describe one of the tests to test overidentifying restrictions. How could you interpret the rejection of the null hypothesis?

3. (25/100) Consider the following AR(1) process:

$$u_t = \rho u_{t-1} + \epsilon_t \qquad \epsilon_t \sim iid(0, \sigma_\epsilon^2) \qquad |\rho| < 1 \qquad t = 1, \dots, n. \tag{3}$$

(a) (4/100) Write down explicitly the form of $\Omega(\rho)$, the covariance matrix of the vector u.

(b) (4/100) Consider the model $y = x(\beta) + u$, with $E(uu') = \Omega$. Write down the GNLS criterion function and demonstrate that F.O.C for minimization are equivalent to the moment conditions.

(c) (2/100) Consider again the model specified in (b). Write down GNR for this model.

(d) (15/100) Imagine you do not know whether u is homoskedastic or heteroscedastic. How would you test it?

4. (25/100) Given is the model

$$y_t = X_t\beta + u_t, \qquad t = 1, \dots, n \tag{4}$$

where $E(uu') = \Omega$ is an $n \times n$ covariance matrix, and X is an $n \times k$ matrix of regressors. The disturbances u are not independent of the regressors X, hence use is made of a matrix W of l instruments, with $l \geq k$. For instruments W it is known that $E(u_t/W_t) = 0$, $E(u_t u_s/W_t, W_s) = \omega_{st}$ implying that $var(n^{-1/2}W'u) = \frac{1}{n}E(W'\Omega W)$.

(a) (2/100) Write down the theoretical and empirical moment conditions that lead to GMM estimator.

(b) (2/100) Solve moment conditions in case $l = k$. Which estimator do you get?

(c) (12/100) Consider the case when $l > k$. Select WJ as instruments with $J : l \times k$ of full (column) rank. Demonstrate which is the optimal choice of J.

(d) (3/100) Write down the GMM criterion function. Explain how to test for overidentification. Compare this case with IV.

(e) (6/100) Consider the model (4) with $E(uu') = \sigma^2 I$. Estimate it by GMM using the weighting matrix Λ, which is $l \times l$, positive definite and at least asymptotically nonrandom. Explain how you get consistent and efficient estimator of β in this case.