Exam Advanced Econometrics
Master Econometrics and Operations Research
Vrije Universiteit Amsterdam
Tuesday 24 October 2006
15.15 – 18.00

Important notes:

- This exam consists of 4 numbered pages.
- Answers should be in English.
- Read the whole Questions first, before answering.
- **Motivate all your answers** and computations.
- Describe your computations clearly.

- Use clear notation in your derivations.
- Be **concise** in all your answers.
- You may use a calculator for numerical computations.
- For all tests use a significance level of 5%, unless state otherwise
- For each question the number of points (on a scale of 100) is clearly indicated.
- The questions should be handed back at the end of the exam. You may not take them with you.
- Distribute your time efficiently.

1. (30/100)
Consider the regression model:

$$y = X\beta + u, \quad u \sim N(0, \Sigma) \tag{1}$$

where y is an $(n \times 1)$ vector, X is a predetermined $(n \times k)$ matrix. In addition, assume we have another predetermined $n \times 1$ vector Z.

(a) Assume that $\Sigma = \sigma^2 I_n$. Suppose that this model is potentially subject to r, linear restrictions, which can be written as $\boldsymbol{R}\beta = \boldsymbol{r}$, where $\boldsymbol{R}$ is an $r \times k$ matrix with $r < k$. Rewrite the model so that the restrictions become r zero restrictions. What are your assumptions?

(b) Assume that $\Sigma = \sigma^2 I_n$. We estimate model (1), but the true model is $y = X\beta + Z\gamma + u$. What are the properties of the OLS estimator for β? Motivate your answer.

(c) Assume Σ is an unknown diagonal matrix. The heteroskedasticity has the form

$$\log \sigma_t^2 = Z_t'\alpha = \alpha_1 + \alpha_2 z_{2t},$$

where α is a vector of unknown parameters. Write down the likelihood of this model and derive the score form of the LM test for the hypothesis of homoskedasticity against the given alternative.
What is the asymptotic distribution of the test-statistic?

(d) How would you test the same hypothesis of homoskedasticity via a Gauss Newton Regression? Give details.

2. (35/100)
Consider the stationary regression model

$$y_t = \beta_1 + \beta_2 y_{t-1} + u_t, \qquad u_t \sim NID(0, \sigma^2), \qquad t = 2, \ldots, n. \tag{2}$$

with $0 < \beta_2 < 1$. Figure 1 shows the Empirical Distribution Functions (EDF) of the $t-$test for a specific value of the constant term. The EDFs are based on a Monte Carlo experiment concerning the OLS estimator of the coefficients in (2) with $n = 20$. $B = 500$ replications were generated. Figure 2 shows corresponding EDFs for $t-$tests on specific values for the coefficient of y_{t-1}. Note that different values for β_2 are used in the DGP.

(a) Show that both regressors in model (2) are predetermined.

(b) Given the OLS estimate for β_2, show that the $t-$test statistic for $\beta_2 = 0.8$ is asymptotically normal with zero mean and unit variance when this null hypothesis is true.

(c) Is the $t-$test statistic for $\beta_1 = 0.5$ pivotal for $n = 20$? Is it asymptotically pivotal? Use Figure 1 to motivate a part of your answer.

(d) Does the asymptotic theory of part (b) apply for testing hypotheses on β_2 for the finite sample size $n = 20$? Motivate your answer. Suggest a feasible simulation based test for $\beta_2 = 0.8$ that you can use in empirical research. Give a detailed description of each step of your testing procedure. How would you initialise y_1?

(e) Show how to construct a standard 90 % asymptotic confidence interval and show how to construct a simulation based 90% confidence interval for β_2. Discuss the expected differences between the two confidence intervals if $n = 20$, $\beta_1 \approx 0.5$ and $\beta_2 \approx 0.8$, given the Monte Carlo results in Figure 2.

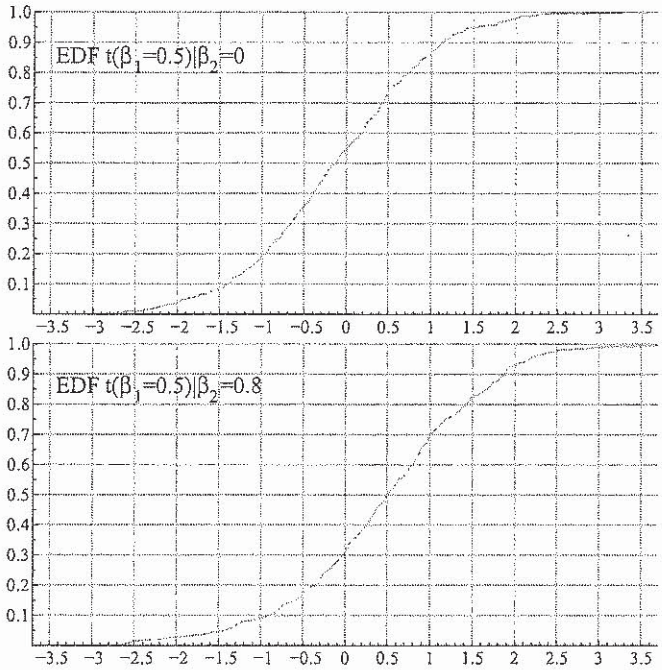

Figure 1: EDF of the OLS based $t-$ statistic for $\beta_1 = 0.5$ in model (2) with $n = 20$, $\beta_1 = 0.5$ and $\sigma = 1$ from a Monte Carlo experiment with 500 replications; Top: DGP with $\beta_2 = 0$, Bottom: DGP with β_2=0.8.

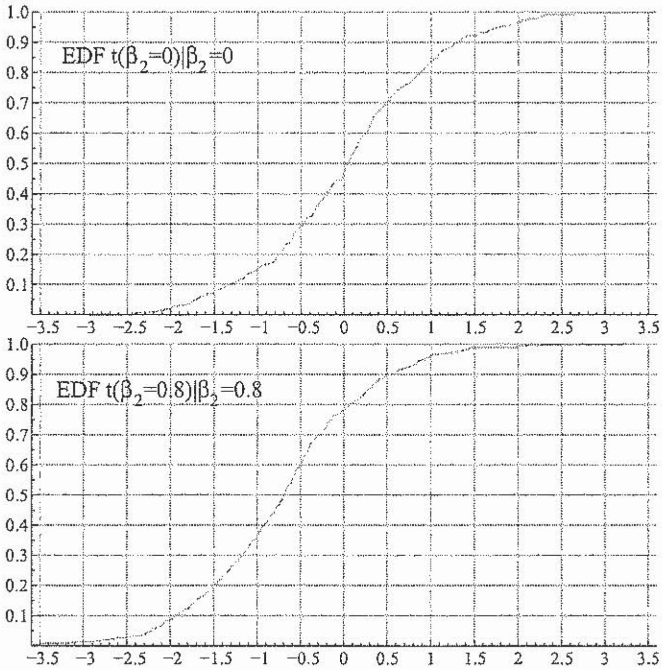

Figure 2: EDF of the OLS based $t-$statistic for $\beta_2 = \beta_{2,0}$ in model (2) with $n = 20$, $\beta_1 = 0.5$ and $\sigma = 1$ from a Monte Carlo simulation with 500 replications;. Top: DGP with $\beta_{2,0} = 0$, Bottom: DGP with $\beta_{2,0}$=0.8.

3. (35/100)

(a) Formulate the Seemingly Unrelated Regression (SUR) model and discuss the corresponding assumptions. How would you estimate the model? Discuss feasible GLS and ML estimation.

(b) In which case(s) does OLS estimation of the SUR model provide efficient estimates? Motivate your answer.

Consider the Simultaneous Equations Model, where the ith equation is given by:

$$y_i = X_i\beta_i + u_i = Z_i\beta_{1i} + Y_i\beta_{2i} + u_i, \quad i = 1, \ldots, g \tag{3}$$

where y_i is an $n \times 1$ vector of the dependent variable i, X_i is an $n \times k_i$ matrix of explanatory variables, that can be partitioned as $[Z_i \;\; Y_i]$. The $n \times k_{1i}$ matrix Z_i is assumed to be predetermined and Y_i is an $n \times k_{2i}$ matrix of endogenous variables. The $n \times 1$ vector error terms u_i are contemporaneously correlated but homoskedastic and serially uncorrelated. Denote the total $n \times l$ instrument matrix by W.

(c) Give the structural form, restricted reduced form and the unrestricted reduced form of the model in (3). Define your matrices properly.

(d) Discuss 3SLS estimation of the Simultaneous Equations Model.

(e) For $g = 2$ discuss FIML and LIML estimation. In your answer, formulate the objective functions of these estimators, discuss a case where FIML and LIML coincide, and discuss the efficiency of these two estimators, where a mathematical derivation of efficiency is not required.