Exam Advanced Econometrics
Master Econometrics & Operations Research
Vrije Universiteit Amsterdam
25 October 2005
15.15 – 18.00

- This exam consists of 3 numbered pages
- You may use a calculator for numerical computations.
- Read the whole Questions first, before answering.
- **Motivate all your answers** and computations.
- Describe your computations clearly.
- Use clear notation in your derivations.
- Be **concise** in all your answers
- Answers should be in English
- For all tests use a significance level of 5%, unless state otherwise
- For each question the number of points (on a scale of 100) is clearly indicated.
- The questions should be handed back at the end of the exam. You may not take them with you.

1. (30/100) Consider the partitioned regression model:

$$y = X\beta + u, \quad u \sim N(0, \sigma^2 I_n) \tag{1}$$

$$= \begin{bmatrix} X_1 & X_2 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} + u, \tag{2}$$

where the matrices and vectors have the following sizes

$$\underset{(n\times 1)}{y}, \quad \underset{(n\times k)}{X}, \quad \underset{(k\times 1)}{\beta}, \quad \underset{(n\times k_1)}{X_1}, \quad \underset{(n\times k_2)}{X_2}, \quad \underset{(k_1\times 1)}{\beta_1}, \quad \underset{(k_2\times 1)}{\beta_2}.$$

X is a matrix of exogenous variables.

(a) Formulate and prove the Frisch-Waugh-Lovell theorem.

(b) Model (1) is estimated by OLS and the residuals are denoted by $\hat{u}_t$. Show that $\text{Var}(\hat{u}_t) < \sigma_0^2$, where σ_0^2 is the true value of σ^2.

(c) Suppose in this question that the true DGP is

$$y = X_1\beta_1 + u \tag{3}$$

However, we estimate model (2) by OLS. Denote the estimator of β_1 by $\hat{\beta}_1$. Is $\hat{\beta}_1$ still unbiased and consistent?

Denote the OLS estimator of β_1 in (3) by $\tilde{\beta}_1$. If we compare $\tilde{\beta}_1$ with $\hat{\beta}_1$, which of the two is more efficient. Or can't we we say anything about it?

(d) Suppose in this question that model (2) is the true DGP, but we estimate model (3) by OLS. What can you say about the properties of the OLS estimator for β_1, i.e. is the estimator unbiased and consistent?

(e) We wish to test for $\beta_2 = 0$. Derive the F-statistic and express it in terms of y. What are the distributions of the nominator and the denominator of the F-statistic?

(f) We wish to calculate the $p-$value of the test $\beta_2 = 0$. Describe a nonparametric bootstrap procedure to obtain the $p-$value.

2. (25/100) Consider an analysis of the model

$$y_t = \beta_1 + \beta_2 x_{1,t} + \beta_3 x_{2,t} + \beta_2\beta_3 x_{3,t} + u_t; \; u_t \sim IID(0, \sigma^2) \tag{4}$$

with Gauss Newton Regressions.

(a) Discuss One-step estimation of the regression parameters of model (4). Hint: Derive first a root-n consistent estimator of the regression parameters.

(b) Discuss iterative estimation of model (4) using Gauss Newton regressions. What's the difference in asymptotic properties if we compare the iterative estimator with the estimator derived under (a)?

(c) Suppose an economic theory suggests that $\beta_2\beta_3 = 1$ in model (4).
Show how to estimate the model using GNR under this restriction.

(d) How can you test the restriction $\beta_2\beta_3 = 1$ by GNR? Describe the test statistic, its asymptotic distribution under the null hypothesis and its critical region. Hint: first rewrite the restriction as a zero restriction, before applying the GNR theory.

3. (25/100) Consider the linear model

$$y = X\beta + u, \quad E(u|X) = 0, \quad E(uu'|X) = \Omega(\gamma_0, \gamma) = \text{diag}\{\exp(\gamma_0 + Z_1\gamma), \ldots, \exp(\gamma_0 + Z_n\gamma)\}, \tag{5}$$

where y is the $n \times 1$ observation vector, X is a $N \times k$ predetermined matrix, β is a fixed unknown $k \times 1$ coefficient vector, $Z = (Z_1', \ldots, Z_n')'$ is a $n \times m$ predetermined matrix, γ_0 is a fixed unknown constant and γ is a fixed unknown $m \times 1$ coefficient vector. Note that $\text{diag}\{x_1, \ldots, x_n\}$ defines a diagonal matrix with scalar x_t as its t-th diagonal element.

(a) What are the asymptotic properties of the OLS estimator of β ?

(b) Assume that γ is known. Consider the method of moment (MM) estimator for β. What is the variance matrix of the MM estimator of β ? Which weighting matrix produces the most efficient estimator ? Motivate your answer.

(c) Assume that γ is unknown. Describe a feasible procedure for the estimation of β, γ_0 and γ of the linear model (5).

(d) Does your procedure in (c) produce consistent estimates ? Motivate your answer.

(e) Describe a specification test for heteroskedasticity in model (5). Provide the asymptotic distribution of your test statistic.

4. (20/100) Consider ML estimation of a model with loglikelihood $l(y^n, \theta) = \sum_{t=1}^{n} l_t(y^t, \theta)$.

(a) Discuss type 1 and type 2 ML estimation, in particular discuss the likelihood equations for type 2 ML estimation.

(b) Discuss the asymptotic distribution of the gradient of the loglikelihood in the context of type 2 ML estimation in the case of independent observations y_t.

(c) Given consistency, given the information matrix equality, given standard regularity assumptions, and assuming correct specification of the model, derive the asymptotic normality of the type 2 ML estimator $\hat{\theta}$. Compute the rate of convergence and the asymptotic covariance matrix of $\hat{\theta}$. Hint: use a proper Taylor expansion of the likelihood equations and scale by proper powers of n.

(d) Suppose the information matrix equality does not hold. How would you derive the asymptotic covariance matrix of $\hat{\theta}$ in that case?